Low-Frequency Stabilization for Maxwell's Equations and Applications using Reduced Order Modeling

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The simulation of electronic devices require in many applications robust solutions from DC to high frequencies. For this purpose we introduce a symmetric low-frequency stable formulation of Maxwell's equations which allows the simulations of electrically large structures down to DC. It is discretized in space by the finite element method. The numerical complexity, in particular in multi-query scenarios, is decreased by employing a reduced order modeling method allowing for efficient computations.

***Index Terms*—Maxwell equations, low-frequency, frequency domain, multi-query, robust formulation**

I. Introduction

SINCE the complexity of electronic devices in industrial microwave engineering keeps growing, modern design processes rely on numerical simulation for creation, modification, analysis, or optimization. The computer-aided design (CAD) process of such devices requires accurate and fast simulations. The numerical analysis of electromagnetic structures of large scale necessitates the solution of partial differential equations (PDEs) stemming from Maxwell's Equations.

In the literature, e.g., [1], [2], the use of the $\mathbf{E}$-field formulation in frequency domain is suggested for fast frequency sweeps, e.g., for the evaluation of a broad-band transfer function (e.g., S-parameters). The formulation reads

$$\operatorname{curl} \mu^{-1} \operatorname{curl} \mathbf{E}(\omega) + (i\omega\sigma + (i\omega)^2\varepsilon)\mathbf{E}(\omega) = -i\omega\mathbf{J}(\omega), \quad (1)$$

where ε denotes the permittivity, σ the conductivity, and μ the permeability. $\mathbf{E}(\omega)$ is the electric field and the magnetic field is obtained through $\mathbf{H}(\omega) = -1/(i\omega\mu) \operatorname{curl} \mathbf{E}(\omega)$; $\mathbf{J}(\omega)$ is a given source current. We assume linear scalar valued materials, and for simplification of the notation perfect electric conducting (PEC) boundary conditions. Yet, the formulation easily extends to perfect magnetic conducting (PMC) and surface impedance boundary conditions and dispersive materials. However this formulation suffers from stability issues as the frequency ω tends to zero, which is discussed for instance in [3].

The simulation of the device as part of an electrical network and time domain computations require a robust solution in a frequency range from DC to high frequency for many applications (e.g. the computation of transient $\mathbf{E}$-, $\mathbf{H}$-, and far-fields). To this end several potential-based $\mathbf{A}$-Φ-formulations have been proposed in literature [4], [5], [6], [7], to account for the lack of stability in the $\mathbf{E}$-field formulation. Based on these works, a new symmetric low-frequency stable formulation is proposed which does not require supplementary equations in comparison to (1) and, additionally, can handle excitations by non-solenoidal currents (the closure of the current loop is implicitly performed by the solution). Its finite element (FE) discretization yields the same number of equations as (1). This is not the case for certain $\mathbf{A}$-Φ-formulations.

Regardless of the formulation, each FE discretization yields a high-dimensional system of linear equations, which consequently requires substantial numerical effort to be solved. A reduced order modeling (ROM) method is applied in order to obtain a reduced order model which still preserves the frequency dependency of the original system, but of much lower dimension. Thus it allows for a fast evaluation of suitable frequency sweeps, cf. Fig. 1.

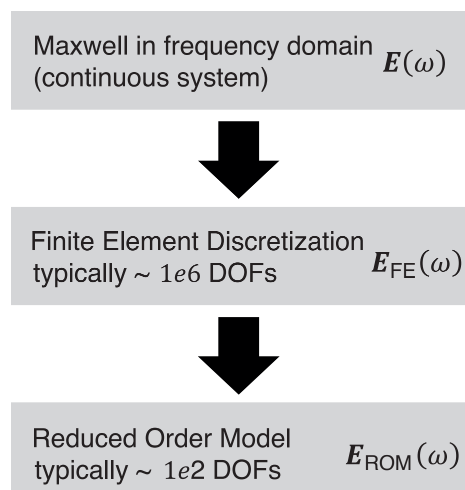

Fig. 1. Model order reduction scheme. Typically the factor in dimension between the FE and the ROM system is of a couple of orders of magnitude.

II. Low-frequency Stable, Symmetric Formulation

Considering the limit as $\omega \to 0$ in the $\mathbf{E}$-field formulation (1) one observes that the system is described by the curl-curl term. However, gradient fields lie in its kernel and thus these components (arising, e.g., as solutions to electrostatic behavior) cannot be recovered by solving (1) especially at DC. Using a particular gauging choice and the ansatz $\mathbf{E}(\omega) = i\omega\mathbf{A} + \nabla\Phi$, where $\mathbf{A}$ and $\nabla\Phi$ live on disjoint subspaces of the same function space as $\mathbf{E}$ previously, for the electric field we derive a stable formulation for Maxwell's equations. In the FE-discretized setting the gauging condition can be realized by using a tree-cotree splitting [8], [9]. For simplicity of notation we present here the non-conductive case only, i.e., $\sigma \equiv 0$. One obtains the following set of equations

$$\begin{aligned}(\mu^{-1}\operatorname{curl}\mathbf{A}, \operatorname{curl} v) + i\omega(\varepsilon(i\omega\mathbf{A} + \nabla\Phi), v) &= -(\mathbf{J}(\omega), v),\\ (\varepsilon\nabla\Phi, \nabla\varphi) + i\omega(\varepsilon\mathbf{A}, \nabla\varphi) &= -\frac{1}{i\omega}(\mathbf{J}(\omega), \nabla\varphi).\end{aligned} \tag{2}$$

This system is symmetric and can be solved directly if $\mathbf{J}(\omega)$ is solenoidal in $\omega = 0$. However, in the case of non-solenoidal current excitation the E-field is unbounded, i.e.,

$$|\mathbf{E}| \to \infty \text{ as } \omega \to 0.$$

To this end, we propose the ansatz $\mathbf{E}(\omega) = i\omega\mathbf{A} + \nabla\Phi + \frac{1}{\omega}\nabla\Phi_0$, where Φ_0 accounts for the divergent part of the right-hand side. Then the system (2) above can be evaluated in the required frequency interval and the magnetic field is robustly obtained through $\mathbf{H}(\omega) = \mu^{-1}\operatorname{curl}\mathbf{A}(\omega)$. We proceed analogously in the case that we have conductive domains with $\sigma > 0$ in the computational domain to obtain a symmetric stable formulation. For details we refer to the full paper.

III. Reduced Order Model

A FE discretization of (1) using edge elements yields the linear system

$$\left(\mathbf{K} + i\omega\mathbf{M}_\sigma - \omega^2\mathbf{M}_\varepsilon\right)\mathbf{E}_{\mathrm{FE}}(\omega) = -i\omega\mathbf{J}_{\mathrm{FE}}(\omega), \tag{3}$$

where $\mathbf{K}$ denotes the curl-curl matrix, which is symmetric positive semidefinite. $\mathbf{M}_\varepsilon$, $\mathbf{M}_\sigma$ are material mass matrices, which are symmetric positive definite, semidefinite, respectively. Since $\mathbf{K}$ is singular, the stability issues for $\omega \to 0$ are inherited by the ROM system. Thus the discretization and reduction procedure is applied on the symmetric stable formulation (2). This guarantees a low-frequency stable and symmetric FE system with a regular system matrix in $\omega = 0$ and consequently the obtained ROM system is low-frequency stable. For the numerical results we used a block-structure preserving multipoint reduced basis ROM method, cf. [10].

IV. Numerical Results

The stable formulation and a ROM approach, i.e., a block-structure preserving multipoint reduced basis method, was applied for the simulation of a package. Field solutions and S-parameters of the structure were computed; they describe the electrical behavior under excitation by electrical signals. The DC field result is shown in Fig. 2. The full paper features detailed data in terms of computation times and data usage.

V. Conclusion

In this work a low-frequency stable formulation of Maxwell's equations was discussed. We presented a symmetric, stable formulation which allows for describing even electrically large structures accurately down to DC. A ROM method was applied in order to obtain a reliable but fast to evaluate model which describes the structure in the required frequency range. The capability of the proposed method was illustrated with a numerical example focusing on low-frequency stability. The full paper will discuss the formulation in more details, in particular the cases of non-solenoidal current excitations and lossy materials, e.g., Ohmic losses. Numerical examples will be discussed and underline the robustness of the stable formulation and the efficiency of the ROM approach.

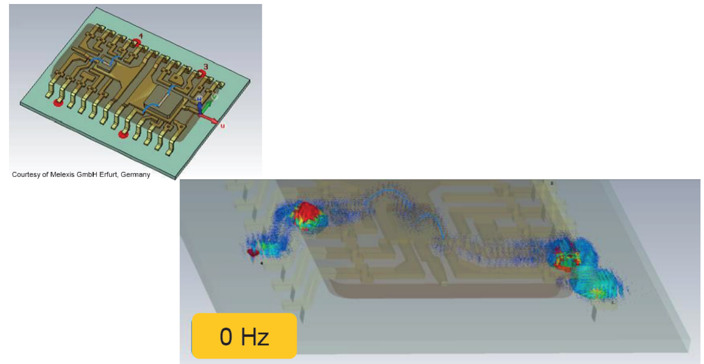

Fig. 2. A package structure. The $\mathbf{E}$-field in 0Hz recovers the DC current distribution, which cannot be obtained through (1).

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